

Stochastic optimization applied to the prepositioning of disaster relief supply decisions in Brazil

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Abstract

We present a two-stage stochastic optimization model to locate pre-positioned materials for disaster relief in Brazil. Due to uncertainty both of disaster severity and media influence, they are represented as scenarios. Results show that the stochastic model generates more robust solutions, particularly when demand cannot be completely fulfilled.

Keywords: humanitarian logistics, facility location, stochastic optimization

Introduction

The increase in the number of people affected by natural (hurricanes, floods, earthquakes, tsunamis) and anthropogenic disasters (terrorist attack, technological or nuclear accident) has required major efforts of relief organizations and emergency operation teams.

Several recent events have demonstrated the vulnerability of societies, such as the tsunami and the earthquake in the Indian Ocean in 2004 and Japan in 2011, hurricanes in the Caribbean, earthquakes in Pakistan in 2005, China in 2008, in Haiti and Chile in 2010, and in New Zealand in 2011, in Brazil, floods occurred in the Itajai valley in 2008, and São Luiz do Paraitinga in early 2011, in addition to catastrophic landslides in Rio de Janeiro in 2011.

Forecasts estimate that over the next 50 years, natural and man-made disasters will increase fivefold in number and severity (Thomas and Kopczak, 2005). There are also predictions of increased frequency of storms in southeastern Brazil as a result of global warming (FAPESP, 2011), which makes preventive measures necessary, including the pre-positioning of relief supplies.

Relief supplies are basic elements that affected people have access to food and hygiene products whereby in the first moments after the disaster. The agility and readiness in the

distribution of these items are necessary, especially in the first 72 hours after the event, so that rescue teams begin the activities and the victims can thus stabilize their lives. Also included are materials required for relief teams (response) to act immediately after the event.

The importance of pre-positioning relief supplies was demonstrated when Hurricane Katrina devastated New Orleans in 2005. The lack of stored materials and the delay in arrival of these supplies hampered further relief to the victims. Problems with legislation and difficulties in defining responsibilities and authority (federal x state government) caused slow response (Holguin-Veras *et al.*, 2007).

In the network configuration, the strategy for locating, along the humanitarian logistics supply chain, is characteristically relevant to the response time of a disaster (Balcik and Beamon, 2008). Facility location decisions affect the performance of the emergency relief operations in disaster, since the number, location of distribution centers and the amount of supply reliefs therein directly affect the response time and costs observed along the supply chain (Barbosa *et al.*, 2010).

This paper proposes a mathematical model to support decisions of locating relief supplies facilities. An application in Brazil (São Paulo State) illustrates the effectiveness of the proposed approach. Through a two-stage stochastic optimization model (Dantzig, 1955), sites are evaluated for installing distribution centers of these materials. This optimization process results in proposing locations that minimize the operational total cost through opening or not relief supply depots considering opening costs, and penalties for unmet demand. Uncertainty is introduced through demand (defined by the disaster severity and magnitude and media coverage) and accessibility ruptures in some areas (which may lead to inaccessible areas).

Literature review

Location of humanitarian facilities using Stochastic Optimization

Chang *et al.* (2007) use stochastic optimization to determine the location of warehouses for materials inventory, allocation and distribution of resources for rescue in cases of urban floods. Due to uncertainty, the flood problem is formulated as a two-stage stochastic programming model where the first stage minimizes the distances and the second stage performs the allocation of inventory.

Rawls and Turnquist (2010) present a two-stage stochastic model for facility location considering various scenarios that may occur in a disaster, assigning each uncertainty in demand and penalty for unmet demand. Due to the complexity of the problem Lagrangian L-shaped heuristic was used for the solution.

Rawls and Turnquist (2011) used constraints of quality of service and average distance of deposits up to demand nodes, performing an application in the South of the United States. Later, Rawls and Turnquist (2012) adapted the model for dynamic allocation (72 hours in advance) for short-term demands, which ensured meeting 100% customer service needs. The penalties for unmet demand in the papers of Rawls and Turnquist range from 10 to 50 times the value of the product. These values showed that for a given problem situation, the change in value of penalties affects the amount of deposits opened, as well as the total cost, indicating that the subjectivity of this value affects the problem solution.

Noyan (2012) incorporated the risk measurement model, also using two-stage stochastic programming by introducing the concepts of expected value of perfect information (EVPI) and the value of stochastic solution (VSS) in the model structure. The value of the penalty was

established as 10 times (in some cases 5 times) the value of the product. Benders decomposition was used for the model solution. The results showed the importance of the risk allocation in locating humanitarian facilities.

Mete and Zabinsky (2010) evaluated the location of medical supply warehouses and inventory levels required for each medical source (first-stage decision) and delivery requirements of supplies through a second stage vehicle routing, which disaggregates the strategic information in operational planning. The model captures specific information to each disaster and its possible effects through the use of scenarios evaluating preparation, risk and uncertainty of the event.

Salmerón and Apte (2010) propose a two-stage stochastic model in which the decision of the first stage refers to the strategy of locating supply relief facilities and the second stage refers to performing activities of transportation necessary to serve the population. The objective function minimizes the number of deaths and the scenarios set are the uncertainties about the location and severity of the event.

Bozorgi-Amiri *et al.* (2011) developed a robust stochastic multiobjective programming for logistics in emergency relief environment under uncertainty. In their approach, not only demand, but also the costs of supplies, the acquisition process and transport are considered as uncertain parameters, there is also the possibility of a disruption of one of the deposits. The objective function minimizes the total cost and penalizes the unmet demand.

Murali *et al.* (2012) consider a problem of locating capacitated facilities to determine points where medicines against a hypothetical anthrax attack in Los Angeles would be delivered to the population. A special case is formulated as a maximum coverage model and decides the locations facilities would be open, and the supply quantity assigned to each location, considering uncertainty in demand. The results compare solutions using heuristics location-allocation and simulated annealing metaheuristic. For a quantity of 40 facilities to be opened, the location-allocation heuristic performed (89.66%) better regarding coverage compared to simulated annealing (82.45%).

A bi-objective model with stochastic demand was formulated by Tricoire *et al.* (2012). The objectives are given by (i) costs of opening distribution centers and distribution to the demand points and (ii) the unmet demand. To solve the integer programming problem, a branch and cut heuristic was used. Real data application in Senegal showed the viability of the approach.

Zhang *et al.* (2012) approach the issue of secondary disasters that occur after a major natural disaster. Examples of these disasters may be cited as the events of Tōhoku, Japan, in 2011, where a nuclear accident occurred after a disaster of seismic origin. Stochastic demands for the first and second disaster were addressed in an individualized manner with different probabilities for each case. The objective function minimizes the rescue costs.

Nolz *et al.* (2011) formulated a multiobjective optimization problem in the design of a logistics system to ensure the adequate distribution of emergency assistance after natural disasters, when damage to infrastructure can interrupt the delivery of humanitarian aid. The problem is formulated encompassing three objective functions and solved using a genetic algorithm. The first objective function minimizes the risk measure; the second objective function minimizes the sum of the distances between all the inhabitants and their nearest service stations; and the third objective function minimizes the total travel time.

Comparing the stochastic solution

Noyan (2012) highlights that the EVPI - Expected Value of Perfect Information and the VSS - Value of the Stochastic Solution (Birge and Louveaux, 1997) are the two best-known

performance measures of stochastic solution, however, not all cited papers use these measures to evaluate the stochastic solution. Table 1 shows how these evaluation measures are approached in the humanitarian logistics literature.

Table 1 - Performance evaluation of stochastic models

Author	Solution performance evaluation
Chang <i>et al.</i> (2007)	Compare the costs of the stochastic, deterministic and current (mean and standard deviation) solution. Difference between stochastic and deterministic = 0.647%.
Rawls and Turnquist (2012)	Does not address the performance of the solution. The solution was complemented by Noyan (2012),
Mete and Zabinsky (2010)	Used the deterministic and the stochastic solution, without establishing a comparison.
Salmerón and Apte (2010)	EVPI between 24% and 25% of the wait-and-see solution, mean VSS between scenarios = 47% of wait-and-see. In the best-case scenario VSS= 256%.
Bozorgi-Amiri <i>et al.</i> (2011)	Compare deterministic solutions with stochastic solution - average gain = 3.8%.
Murali <i>et al.</i> (2012)	Focus on the solution methodology.
Tricoire <i>et al.</i> (2012)	Does not address the performance of the solution.
Zhang <i>et al.</i> (2012)	Focus on algorithm, 26.4% gain over time.
Noyan (2012)	EVPI 54.05% to 58.42%, VSS 0.84% to 5.41% of wait-and-see solution.
Nolz <i>et al.</i> (2011)	Performs a sensitivity analysis on the basis of the change in risk.

The mathematical model

The goal of the model proposed in this paper is the establishment of local installation of one or more permanent distribution centers for storage of relief supplies aimed at aiding the victims of natural disasters that may occur in a region. The objective function minimizes the total cost of attendance, composed of the costs of opening the warehouse, transportation and penalties for unmet demand. Constraints can be grouped as capacity (storage and transport), available materials (inventory, donations, and purchases) and minimum level service (minimum met demand).

The problem is modeled as two-stage stochastic optimization model and is based on papers presented by Mete and Zabinsky (2010) and Rawls and Turnquist (2011). Figure 1 illustrates the structure of the model:

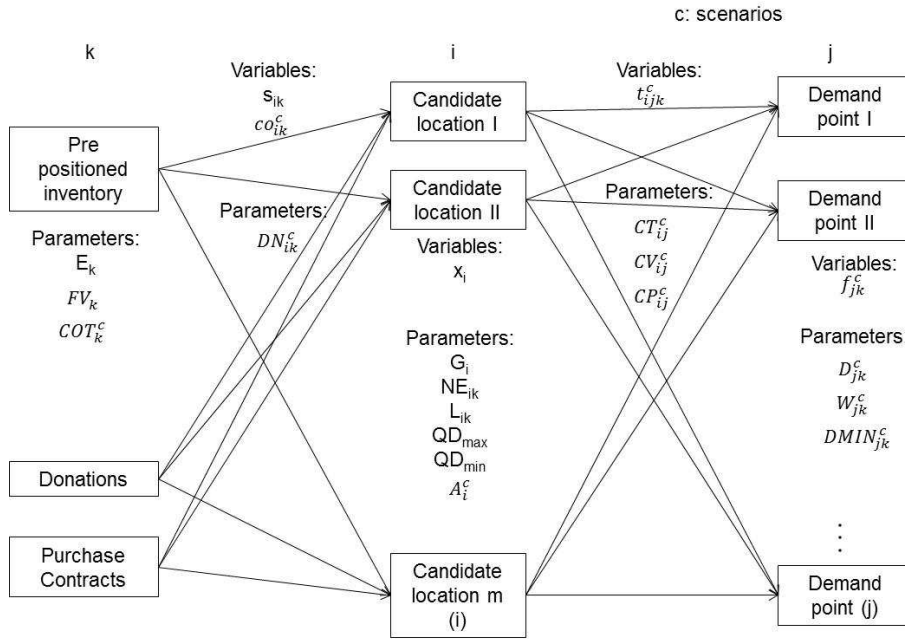


Figure 1 - Model structure

Sets

I: candidate distribution centers ($i \in I$)

K: relief supplies ($k \in K$)

J: demand points ($j \in J$)

C: scenarios ($c \in C$)

First stage decision variables:

x_i : 1 if distribution center i is opened, 0 otherwise

s_{ik} : average inventory level of supply relief k at distribution center i (kg)

Second stage decision variables:

t_{ijk}^c : amount of k to transport from distribution center i to the point of demand j , under scenario c (kg)

f_{jk}^c : unmet demand of k , at point j under scenario c (kg)

co_{ik}^c : amount of k purchased, allocated in distribution center i , under scenario c (kg)

$co_aux_k^c$: auxiliary binary variable to make purchases only if k is necessary

Parameters:

Scenario non-dependent:

G_i : annual cost of installation and operation of distribution center i (\$)

E_k : amount available of supply k (kg)

L_{ik} : maximum storage capacity of k in distribution center i (kg)

NE_{ik} : minimum annual inventory of k in distribution center i (kg)

QD_{max} : maximum number of distribution centers to be opened

QD_{min} : minimum number of distribution centers to be opened

FV_k : weight x volume conversion factor (m^3 / kg)

M : large number for making purchases of supplies k only if necessary

Scenario dependent:

CT_{ij}^c : transportation cost from distribution center i to demand point j under scenario c (\$ / kg)

W_{jk}^c : penalty per unit of k not supplied to demand point j under scenario c (\$ / kg)

DN_{ik}^c : amount of donations of k received in distribution center i under scenario c (kg)

D_{jk}^c : demand of k in demand point j under scenario c (kg)

A_i^c : binary parameter regarding the accessibility of distribution center i (1 - accessible, 0 not accessible) under scenario c

CP_{ij}^c : transportation capacity (by weight) from distribution center i to demand point j under scenario c

CV_{ij}^c : transportation capacity (by volume) from distribution center i to demand point j under scenario c (m³)

$DMIN_{jk}^c$: minimum demand of k to be supplied at demand point j , under scenario c (kg)

COT_k^c : contractual limit established for purchases of k , under scenario c (kg)

First stage objective function:

Minimize the [(operating cost of distribution centers) + (expected value of the solution of the second stage function)]

$$\min \sum_i G_i x_i + E_c[Q(x, s, c)] \quad (1)$$

First stage constraints:

Constraint (2) establishes that, for an item k , the amount stored at every distribution center can not exceed the maximum amount available.

$$\sum_i s_{ik} \leq E_k \quad \forall k \in K \quad (2)$$

Constraint (3) limits the inventory level by the capacity of distribution center i .

$$L_{ik} x_i \geq s_{ik} \quad \forall i \in I, k \in K \quad (3)$$

Constraint (4) limits the minimum inventory of the item k to open a distribution center i .

$$NE_{ik} x_i \leq s_{ik} \quad \forall i \in I, k \in K \quad (4)$$

Constraint (5) and (6) defines the maximum and minimum number of distribution center to be opened.

$$\sum_i x_i \leq QD_{max} \quad \forall i \in I \quad (5)$$

$$\sum_i x_i \geq QD_{min} \quad \forall i \in I \quad (6)$$

Second stage objective function:

Minimize [(transportation cost under scenario c + penalty for unmet demand under scenario c)]

$$Q(x, s, c) = \min \sum_i \sum_j \left(CT_{ij}^c \sum_k t_{ijk}^c \right) + \sum_j \sum_k W_{jk}^c f_{jk}^c \quad (7)$$

Second stage constraints:

Constraint (8) ensures that the relief supply k to be transported from i to demand point j is available at i

$$\sum_j t_{ijk}^c \leq s_{ik} + DN_{ik}^c + co_{ik}^c \quad \forall i \in I, k \in K, c \in C \quad (8)$$

Constraint (9) calculates the unmet demand of k in j under scenario c

$$f_{jk}^c = D_{jk}^c - \sum_i t_{ijk}^c A_i^c \quad \forall j \in J, k \in K, c \in C \quad (9)$$

Constraint (10) ensures that the relief supply k to be transported from i to demand point j is at the distribution center opened by x_i

$$L_{ik} x_i \geq \sum_j t_{ijk}^c A_i^c \quad \forall i \in I, k \in K, c \in C \quad (10)$$

Constraint (11) ensures the transport capacity by weight of supply k

$$\sum_k t_{ijk}^c \leq CP_{ij}^c \quad \forall i \in I, j \in J, c \in C \quad (11)$$

Constraint (12) ensures the transport capacity by volume of supply k

$$\sum_k t_{ijk}^c \times FV_k \leq CV_{ik}^c \quad \forall i \in I, j \in J, c \in C \quad (12)$$

Constraint (13) ensures that a minimum demand of k at the demand point j is met.

$$\sum_i t_{ijk}^c A_i^c \geq DMIN_{jk}^c \quad \forall j \in J, k \in K, c \in C \quad (13)$$

Purchases:

Constraint (14) establishes a condition for purchasing relief supplies k : $co_aux = 0$ if Demand - Inventory - Donations > 0

$$(1 - co_aux_k^c) M \geq \sum_j D_{jk}^c - \sum_i s_{ik} - \sum_i DN_{ik}^c \quad \forall k \in K, c \in C \quad (14)$$

Constraint (15) defines when no purchase is requested: $co_aux = 1$ if Inventory + Donations - Demand > 0

$$co_aux_k^c M \geq \sum_i s_{ik} + \sum_i DN_{ik}^c - \sum_j D_{jk}^c \quad \forall k \in K, c \in C \quad (15)$$

Constraint (16) defines purchase of relief supply k only if $co_aux = 0$

$$co_{ik}^c \leq (1 - co_aux_k^c) M \quad \forall i \in I, k \in K, c \in C \quad (16)$$

Constraint (17) ensures that the purchase of supplies k is allocated to the distribution center opened by x_i

$$COT_k^c x_i \geq co_{ik}^c \quad \forall i \in I, k \in K, c \in C \quad (17)$$

Constraint (18) ensures total purchase of supply k allocated to each distribution center i does not exceed the contractual total amount under scenario c .

$$COT_k^c \geq \sum_i co_{ik}^c \quad \forall k \in K, c \in C \quad (18)$$

Constraint (19) ensures that the purchase of supplies k is performed only after the consumption of inventory and the donation received in i .

$$\sum_i co_{ik}^c \leq \sum_j D_{jk}^c - \sum_i s_{ik} - \sum_i DN_{ik}^c + co_aux_k^c M \quad \forall k \in K, c \in C \quad (19)$$

Constraint (20) and (21) defines non-negativity and binary variables, respectively.

$$s_{ik}, t_{ijk}^c, f_{jk}^c, co_{ik}^c \geq 0 \quad \forall i \in I, j \in J, k \in K, c \in C \quad (20)$$

Binary

$$x_i, co_aux_k^c \in \{0,1\} \quad \forall i \in I, k \in K, c \in C \quad (21)$$

Case study

The proposed optimization model is applied to a case study in the Sao Paulo State (Brazil) to evaluate the techniques used and the results. The region was chosen because of the historical data and geographic information available, and mainly because it is a region prone to natural disasters, as in recent events in the cities of Queluz (2000) and São Luiz do Paraitinga (2010).

Five local candidates to distribution center location are considered: São Paulo, Caçapava, São José dos Campos, Taubaté, and Tremembé. These sites were chosen because they already have Civil Defense operations and are situated in locations with a history of few accidents, thus less likely to rupture.

The scenarios:

The scenarios were established according to the severity and magnitude of disasters (medium, large, and catastrophe). In addition, the disclosure in the media was considered at three levels (low, medium, or large). The media plays a key role in a disaster, especially in mobilizing volunteers and donations (Arnold, 2011). Another consideration is possible disruptions that may affect the accessibility of supply channels to affected sites, changing costs of transport and supply. Table 2 shows the probability of scenarios

Table 2 - Probability of scenarios

	Medium	Large	Catastrophe
Low dissemination by media	0.2	0.02	0.01
High dissemination by media	0.3	0.1	0.3
High dissemination by media and ruptures	0	0.01	0.06

Results and discussion

The model was implemented using the software AIMMS 3.11, CPLEX solver 12.3 in CPU Intel Core i3® 2310M 2.1 GHz, 4 Gb RAM, 64-bit operational system Windows7 ®.

Table 3 shows the results of deterministic and stochastic models. The deterministic solution was obtained using the weighted average of the parameters to a 5-year horizon and a value of penalty equal to 5 times the value of the freight. The cost of penalties is the largest component of the total cost. This finding is due to the lack of materials. Even in the deterministic model, where the reduction of unmet demand occurs due to the absence of random parameters (uncertainty), this cost is high which means that the current Civil Defense operation are not adapted even if to the average disaster level. In the stochastic solution, as well as in deterministic solution, the values obtained show that the penalties strongly influence the results due to unmet demand.

Table 3 – Results of the deterministic and stochastic models

	Deterministic (R\$)	Stochastic (R\$)
Fixed Cost deposits open	63,000.00	18,000.00
Transportation costs	30,664.66	25,065.72
Penalties costs	200,381.53	331,532.33
Total cost	294,046.19	374,598.06
Distribution centers opened	São Paulo	São Paulo
	Taubaté	

Could also be observed, by the results, that only in medium disaster could the demand be met, although donations and purchased materials, totaling 40.4% (by weight) of total demand, were not fully used due to capacity constraints of deposits. Another significant cost is fixed cost for opening deposits. The lowest relative cost is the cost of transportation.

Comparison of deterministic and stochastic solutions

The EVPI and VSS values were calculated and compared with the results by Noyan 54.05% to 58.42 for EVPI and 0.84% to 5.41% for VSS and the results achieved by Salmerón and Apte (2010) who obtained EVPI between 24% and 25% and VSS s between 47% (percentage values relative to wait-and-see solution). Considering that smaller EVPI indicates a better solution and higher VSS indicates a better solution, and that the VSS value depends on the value of the penalties, the model shows good results for EVPI, despite requiring a refinement of of the criteria adopted for values of penalties, which would provide an improvement in the value of VSS.

Conclusions

This paper presented a problem of repositioning of disaster relief supply decisions in Brazil through stochastic modeling. The results show that the existing infrastructure in São Paulo state is not able to support the demand of a very large catastrophic disaster. The stochastic modeling shows that the main component is the penalty costs, consequently, the outcome of the model is extremely sensitive to this value. The results suggest that only one distribution center is used to supply relief supplies. The distribution center currently existing in the city of São Paulo

would be used for this purpose, however, in case of disruptions in access to the deposit, another site is needed.

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